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EFFICIENCY OF ESTIMATION WHEN THERE IS ONLY ONE COMMON FACTOR

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EFFICIENCY OF ESTIMATION WHEN THERE IS ONLY ONE COMMON FACTOR*

1. Introduction

Under multinormality assumptions, Lawley (1967) and Lockhart (1967) give formulas for the inverse of the asymptotic variance-covariance matrix of the maximum likelihood estimators for factor analysis parameters. Anderson & Rubin (1956, eq. 12.25) give a complicated formula for the asymptotic covariance between two estimated loadings on the same factor. Explicit formulas for other asymptotic sampling variances and covariances do not seem to be readily available. The present paper gives reasonably convenient explicit formulas for these for the special case where there is just one common factor. (In this paper, the terms asymptotic and large-sample imply a large number of observations, not a large number of variables.) It is shown how n, the number of random variables, affects the efficiency with which factor loadings can be estimated.

When the one-factor model holds for a set of n variables, the covariance σ_{ij} between any two variables x_i and x_j can be estimated more efficiently than by using the ordinary sample covariance s_{ij}. An expression for the increase in efficiency is given. A practical test-theory problem that motivated the derivation is briefly considered in the final section.

2. Formulas for Sampling Variances and Covariances

The factor analysis model for the one-factor case is

$$\Sigma = \lambda\lambda' + \Psi \qquad (1)$$

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where $\Sigma \equiv \|\sigma_{ij}\|$ is the (population) variance-covariance matrix for $n \geq 3$ observed variables x_i, $\lambda \equiv \{\lambda_i\}$ is the vector of loadings on their common factor, and $\Psi \equiv \|\psi_{ij}\|$ is a diagonal matrix. It will be assumed that $\psi_{ii} > e > 0$, $i = 1,2,\ldots,n$, where e is some small quantity.

For the one-factor case, ignoring terms of order $1/N$, the expected values of the second derivatives of the logarithm of the likelihood function may be summarized in matrix form as follows

$$A \equiv -N^{-1}\|\mathcal{E}(\partial^2 \log L/\partial\lambda_i\partial\lambda_j)\| = c\Sigma^{-1} + \Sigma^{-1}\lambda\lambda'\Sigma^{-1} \quad , \tag{2}$$

$$B \equiv -N^{-1}\|\mathcal{E}(\partial^2 \log L/\partial\lambda_i\partial\psi_{jj})\| = \Sigma^{-1}\Delta \quad , \tag{3}$$

$$D \equiv -N^{-1}\|\mathcal{E}(\partial^2 \log L/\partial\psi_{ii}\partial\psi_{jj})\| = \tfrac{1}{2}\|(\sigma^{ij})^2\| \quad , \tag{4}$$

where N is the number of observations on which each σ_{ij} is based,

$$c \equiv \lambda'\Sigma^{-1}\lambda \quad , \tag{5}$$

Δ is the diagonal matrix whose nonzero elements are $(1 - c)\psi^{ii}\lambda_i$, and σ^{ij} and ψ^{ij} are the elements of Σ^{-1} and Ψ^{-1} respectively (in this section, except for N, the number of observations, upper-case letters are used for matrices, lower-case unsubscripted Greek letters for vectors, and lower-case Roman or subscripted Greek letters for scalars). These three equations are readily found from Jöreskog's convenient summary (1969, eqs. 17, 19, 22).

Denote the maximum likelihood estimators of the parameters by $\hat{\lambda}_1$, $\hat{\lambda}_2$, ..., $\hat{\lambda}_n$, $\hat{\psi}_{11}$, $\hat{\psi}_{22}$, ..., $\hat{\psi}_{nn}$. The asymptotic variance-covariance matrix of these estimators is given by the inverse of the

partitioned matrix

$$N\begin{bmatrix} A & B \\ B' & D \end{bmatrix} \quad .$$

This inverse can be found from the standard formula

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BM^{-1}CA^{-1} & -A^{-1}BM^{-1} \\ -M^{-1}CA^{-1} & M^{-1} \end{bmatrix} , \tag{6}$$

where $M \equiv D - CA^{-1}B$.

A standard formula applied to (1) gives

$$\Sigma^{-1} = \Psi^{-1} - (1 - c)\kappa\kappa' \quad , \tag{7}$$

where $\kappa \equiv \Psi^{-1}\lambda$. Similarly, from (2),

$$A^{-1} = c^{-1}(\Sigma - \tfrac{1}{2}\, c^{-1}\lambda\lambda') \quad . \tag{8}$$

From (7), we find that

$$\Sigma^{-1}\lambda = (1 - c)\kappa \quad . \tag{9}$$

From (3) through (9),

$$M \equiv D - B'A^{-1}B = \tfrac{1}{2}\,\|\chi^{ii} + f^{-2}\kappa_i^2\kappa_j^2\| \tag{10}$$

where

$$f \equiv \sum_{i=1}^{n} \psi^{ii}\lambda_i^2 = \frac{c}{1 - c} \quad , \tag{11}$$

$$\chi^{ii} \equiv (f\psi^2_{ii}g_i)^{-1} \quad , \tag{12}$$

$$g_i \equiv (f - 2\psi^{ii}\lambda^2_i)^{-1} \quad . \tag{13}$$

Then,

$$M^{-1} = 2(\chi - d\chi\gamma\gamma'\chi) \tag{14}$$

where $\chi \equiv \|\chi_{ii}\| \equiv \|\chi^{ii}\|^{-1} = \|f\psi^2_{ii}g_i\|$ is a diagonal matrix, $\gamma \equiv \{\kappa^2_i\}$, and $d \equiv (f^2 + \gamma'\chi\gamma)^{-1}$.

We have

$$\chi_{ii}\gamma_i = f\lambda^2_i g_i \tag{15}$$

so that

$$d = [f^2 + f \sum_{i=1}^{n} g_i(\psi^{ii}\lambda^2_i)^2]^{-1} \quad . \tag{16}$$

Finally, after substituting (15) and (16) into (14), we have by (6) the asymptotic variances and covariances of the ψ_{ii} :

$$\mathrm{Cov}(\hat{\psi}_{ii}, \hat{\psi}_{jj}) = \frac{2}{N}(f\psi^2_{ij}g_i - df^2\lambda^2_i\lambda^2_j g_i g_j) \quad . \tag{17}$$

Similarly, after some algebra,

$$\mathrm{Cov}(\hat{\lambda}_i, \hat{\psi}_{jj}) = \frac{1}{N}(-2\lambda_i\psi_{ij}g_i + df^2\lambda_i\lambda^2_j g_i g_j) \quad , \tag{18}$$

$$\mathrm{Cov}(\hat{\lambda}_i, \hat{\lambda}_j) = \frac{1}{N}[\psi_{ij}(1 + g_i) + \frac{1}{2}\lambda_i\lambda_j(1 - df^2 g_i g_j)] \quad . \tag{19}$$

$$\chi^{ii} \equiv (f\psi^2_{ii}g_i)^{-1} \quad , \tag{12}$$

$$g_i \equiv (f - 2\psi^{ii}\lambda^2_i)^{-1} \quad . \tag{13}$$

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$$\mathrm{Cov}(\hat{\lambda}_i,\hat{\lambda}_j) = \frac{1}{N}\,[\psi_{ij}(1 + g_i) + \tfrac{1}{2}\lambda_i\lambda_j(1 - df^2 g_i g_j)] \quad . \tag{19}$$

3. The Role of n

Now that these formulas are available, we can examine the role of n, the number of variables, in determining the sampling variances and covariances. Let us assume that there is a uniform upper bound to all the σ_{ii}. Then it appears from (11), (13), (16) that

f is of order n,

g_i is of order n^{-1},

d is of order n^{-1},

$fg_i = 1$ plus terms of order n^{-1}.

Now, the second term in (17), also both terms in (18), are of order n^{-1}. Thus the sampling covariances between $\hat{\psi}_i$ and $\hat{\psi}_j$, $i \neq j$, also between $\hat{\psi}_i$ and $\hat{\lambda}_i$ or $\hat{\lambda}_j$, vanish for large n. If we neglect terms of order n^{-1},

$$\text{Var } \hat{\psi}_{ii} \doteq 2\psi_{ii}^2/N \quad , \tag{20}$$

$$\text{Var } \hat{\lambda}_i \doteq (\psi_{ii} + \lambda_i^2/2)/N \quad , \tag{21}$$

$$\text{Cov}(\hat{\lambda}_i, \hat{\lambda}_j) \doteq \lambda_i\lambda_j/2N \qquad (i \neq j) \quad . \tag{22}$$

4. Estimating the Covariances between Variables

Under the one-factor model, the maximum likelihood estimators of the population variances and covariances are given by

$$\hat{\sigma}_{ii} \equiv \hat{\lambda}_i^2 + \hat{\psi}_{ii} = \frac{1}{N} \sum_{a=1}^{N} (x_{ia} - \bar{x}_i)^2 \quad , \tag{23}$$

$$\hat{\sigma}_{ij} \equiv \hat{\lambda}_i \hat{\lambda}_j \quad \text{if} \quad i \neq j \quad . \tag{24}$$

Only the second formula gives a different estimator than would be appropriate without the single-factor assumption.

The asymptotic variance of $\hat{\sigma}_{ij}$ is given by

$$\text{Var}\ \hat{\sigma}_{ij} \doteq \lambda_i^2 \text{Var}\ \hat{\lambda}_j^2 + \hat{\lambda}_j^2 \text{Var}\ \hat{\lambda}_i^2 + 2\lambda_i\lambda_j \text{Cov}(\hat{\lambda}_i, \hat{\lambda}_j) \quad .$$

By (21) and (22), omitting terms of order $1/n$, when $i \neq j$

$$\text{Var}\ \hat{\sigma}_{ij} \doteq \frac{1}{N} (\lambda_i^2 \psi_{jj} + \lambda_j^2 \psi_{ii} + 2\lambda_i^2 \lambda_j^2)$$

$$\doteq \frac{1}{N} (\sigma_i^2 \sigma_j^2 + \sigma_{ij}^2 - \psi_{ii}\psi_{jj}) \quad . \tag{25}$$

Without the one-factor assumption, one would estimate σ_{ij} by the second bivariate sample moment

$$s_{ij} \equiv \frac{1}{N} \sum_{a=1}^{N} (x_{ia} - \bar{x}_i)(x_{ja} - \bar{x}_j) \quad , \tag{26}$$

which has an asymptotic variance of $(\sigma_i^2\sigma_j^2 + \sigma_{ij}^2)/N$. Thus, for large n and N the use of the one-factor assumption decreases the sampling variance of our estimate of σ_{ij} by the amount $\psi_{ii}\psi_{jj}/N$.

5. Estimating a Validity Coefficient of a Composite

Suppose x_n is a criterion variable of interest, and suppose that we are interested in using the "total score"

$$X \equiv \sum_{i=1}^{n-1} x_i$$

to predict x_n. The "validity coefficient" for the effectiveness of the total score for this purpose is the correlation

$$\rho(X,x_n) = \sum_{i=1}^{n-1} \sigma_{in}/\sigma_X\sigma_{nn} \quad .$$

When the single-factor model holds, every term in the numerator can be estimated more accurately by (24) than by (26). This seems at first sight to guarantee a reduction in the sampling errors of the estimated validity coefficient, an important consideration when choosing among several predictors.

Some algebra (not given here) shows, however, that the sampling variance of the estimated numerator is the same whether the one-factor model is assumed or not. The same is true for the estimated $\rho(X,x_n)$. The reason is that although Var $\hat{\sigma}_{ij}$ is smaller under (24) than under (26), $\mathrm{Cov}(\hat{\sigma}_{ij},\hat{\sigma}_{gh})$ is larger under (24). Both Var $\hat{\sigma}_{ij}$ and $\mathrm{Cov}(\hat{\sigma}_{ij},\hat{\sigma}_{gh})$ are involved in the formulas for $\mathrm{Var}(\sum_{i=1}^{n-1} \hat{\sigma}_{in})$ and Var $\hat{\rho}(X,x_n)$.

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